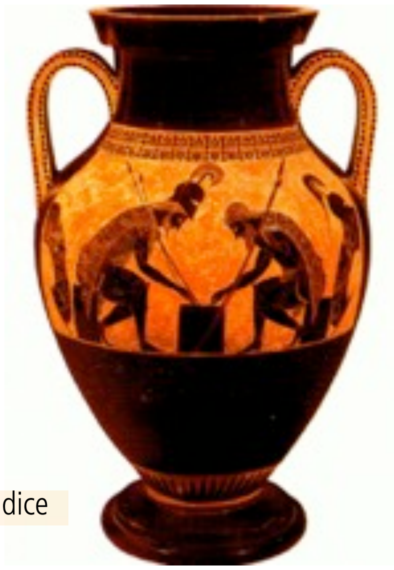
THREE FAMOUS PROBLEMS

Besides the problème des parties in workshop 1 there are many more problems influencing the development of probability calculus. Most of them originate in gambling.

1. AN ANTIQUE PROBLEM

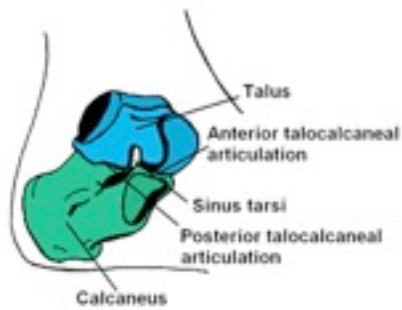
Already in the Egyptian, Greek and Roman culture gambling has been very popular. In one variety of a game of chance, 4 astragals were rolled in one throw. If all the bones showed different sides the throw was called "Throw Of Venus" and won immediately.



Greek amphora, 6th century BC, Achilles and Ajax at dice

The **talus** bone or **astragalus** is a bone in the (...) ankle joint (...). (...) it transmits the entire weight of the body to the foot.
e.wiktionary.org/wiki/anklebone, 2010

In ancient times astralagi of sheep were used as dice. They could lie in 4 different positions. Experiments show the probabilities of each side:



A: 1/10 B: 1/10 C: 2/5 D: 2/5

1 Evaluate the probability of a "Throw Of Venus"

a) with experiments. Simulate 20 throws of four astragali by creating random integers between 1 and 10 with the calculator:
$1 \rightarrow A$, $2 \rightarrow B$, $3-6 \rightarrow C$, $7-10 \rightarrow D$.

b) by calculation.

2. A RENAISSANCE PROBLEM

The first book about probability, the "Liber de Ludo Aleae", was written in 1563 by Girolamo Cardano and originates in his passion for gambling. It is in this book that for the first time the probabilities for the sum of the pips when throwing two dice are calculated correctly:

sum of pips	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
probability	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

Most of the contemporary gamblers thought the chances for e.g. a 9 were exactly the same as for a 10 because both the sums could be achieved in two ways:
$9 = 3+6 = 4+5 \qquad 10 = 4+6 = 5+5$.
They were bewildered by the fact that while gambling 9 appeared more often than 10.

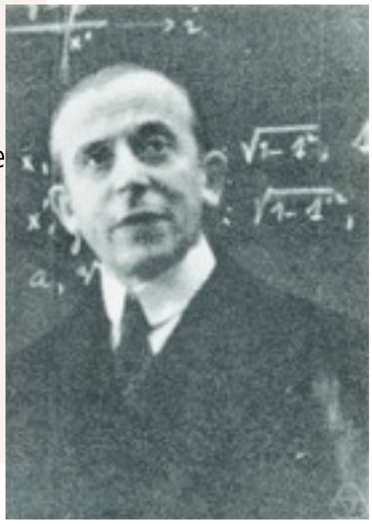
Girolamo Cardano (1501 - 1576), physician, philosopher, technician, mathematician, born in Pavia, lived and worked in Pavia, Milan, Bologna and Rome.
He wrote the most famous book of his time about mathematics, the "Ars maga de Regulis Algebraicis" in which he gave formulae to calculate the solutions of polynomial equations of 3rd and 4th degree.

2 Prove theoretically Cardano's list of probabilities above.

3. THE BIRTHDAY PROBLEM

In 1939, a mathematician named Richard von Mises first proposed what is known today as the birthday problem. He wondered, "How many people must be in a room before the probability that some share a birthday, ignoring the year and ignoring leap days, becomes at least 50 percent?"
It became one of the most famous problems with a solution that contradicts "common sense" completely.

Richard von Mises (1883 - 1953), aerodynamicist and mathematician. He was born in Lemburg, then part of Austria-Hungary (now Ukraine) into a Jewish family. His father worked as a railway engineer and held a doctorate of technical sciences in Vienna.
He studied mathematics, physics and engineering in Vienna. During World War I he first became a test pilot, then he lectured in Strass-bourg and developed an aircraft for the Austro-Hungarian army. It was finished 1916 but never saw action.
After the war he lectured in Dresden and Berlin and in 1933, after the rise of the National Socialist party he moved to Istanbul (Turkey), and in 1939 he accepted a professorship of Aerodynamics and Applied Mathematics at Harvard University (USA).
His academic work was widely spread. He published about fluid mechanics and aerodynamics as well as about probability theory. In 1909 he tried to build the first axiomatic model for probability theory which failed but led Kolmogorov to his successful axiomatisation in 1933.



$\boxed{3}$

a) Guess this number. Later on you will be able to compare your guess with the correct solution.
b) Simulate the birthday situation in a room with 30 people by randomly assigning a birthday (= integer $\in \{1, \ldots, 365\}$) to each of them.

Assumed that 23 people are in the room.
Then the event A = "at least 2 people share the same birthday" is of interest.
This event A contains all the outcomes with 2 people with the same birthday, or 3, or 4 or any integer up to 23.
Very often if an event A is described with the words "at least" or "not more than" the complement $\overline{A}$ is much easier to calculate.

$\overline{A}$ = "no two people have the same birthday" = "all birthdays are different"

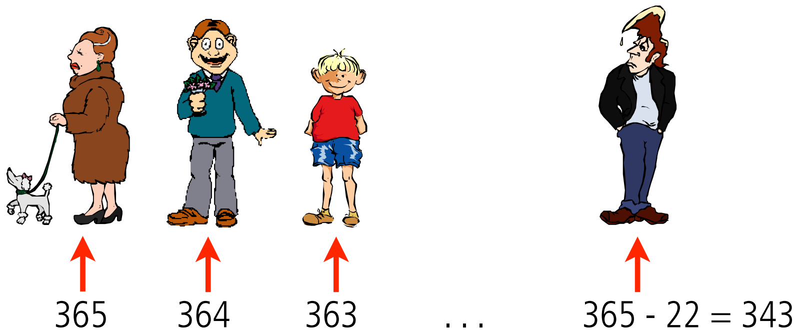
There is a very simple connection between the probability of A and the one of $\overline{A}$:

$$p(A) = 1 - p(\overline{A})$$

So by calculating $\overline{A}$ the probability of A can be calculated.

But what is the probability of $\overline{A}$?

Let us assume the n people are lined up and then to each of them the number of all possible birthdays is assigned:



To get the total of all outcomes of $\overline{A}$ the figures above have to be multiplied:

$$|\overline{A}| = 365 \cdot 364 \cdot 363 \cdot \ldots \cdot 343 \approx 4.22 \cdot 10^{58}$$

The total of all possible outcomes:

$$|\Omega| = 365 \cdot 365 \cdot 365 \cdot \ldots \cdot 365 = 365^{n}$$

Therefore

$$p(\overline{A}) = \frac{\text{number of outcomes in which } \overline{A} \text{ occurs}}{\text{number of possible outcomes}} = \frac{|\overline{A}|}{|\Omega|} = \frac{365 \cdot 364 \cdot \ldots \cdot 343}{365^{n}} \approx 0.49$$

and $\quad p(A) \approx 1 - 0.49 = 0.51 \mathbf{> 0.5}$

With at least **23 people** in a room the chances that at least two of them share the same birthday is greater than 0.5.

4 The problem above often gets confused with the following:
How likely is it that in a class with 23 students at least one more person shares her or his birthday **with me**?